Soliton stability in optical transmission lines using semiconductor amplifiers and fast saturable absorbers

S. K. Turitsyn
Institut für Theoretische Physik I, Heinrich-Heine-Universität Düsseldorf, 40225 Düsseldorf, Germany
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Soliton stability has been examined in the cascaded transmission system based on the standard monomode fibers with in-line semiconductor optical amplifiers, sliding filters, and saturable absorbers (SAs). Stabilization of the pulse propagation in such a system can be achieved under a proper choice of the filter and SA parameters. Conditions of the stable propagation including a critical sliding rate are determined. Impact of the saturable absorber on the soliton stability has been investigated. [S1063-651X(96)50210-2]

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Optical transmission systems using wide-bandwidth semiconductor optical amplifiers (SOAs) operating at 1.3 μm have been put into the focus of intensive studies recently [1–6]. Standard monomode fibers (SMFs) exploiting in most of the installed networks have minimal dispersion at this wavelength. Utilization of in-line SOAs at 1.3 μm in the optical communication systems based on SMFs takes advantage of both operating close to the zero-dispersion point and the wide availability of semiconductor amplifiers. Among the negative factors inherent in such systems should be mentioned the gain saturation, an additional chirp resulting from the SOA action, and the relatively (in comparison with 1.55 μm carrier wavelength) higher value of fiber losses at 1.3 μm. The performance of transmission systems operating at 1.3 μm in links based on SMFs and using in-line SOAs has been studied in Refs. [4–6].

It is an important feature of semiconductor amplifiers that they are quickly saturated and for high bit rate transmission could not recover before the next optical pulse in the pattern reaches the amplifier. A guiding-center soliton that is a natural nonlinear mode of the transmission system with periodic linear amplification is not a stable asymptotic state of the communication system with SOAs. An input pulse in the form of the soliton of the nonlinear Schrödinger equation (NLSE) evolves into a structure that differs significantly from the fundamental soliton. In other words, such a transmission system is perturbed and unstable for an input signal in the form of the fundamental soliton. It should be pointed out, however, that for transmission over several hundred kilometers this instability does not play a crucial role. The instability can be suppressed by the installation of either filters or acousto-optic modulators to achieve long-distance, stable transmission. Use of the sliding filters leads to the stabilization of the pulse propagation [2,3]. It is known for the system with periodic linear (nonsaturated) amplification that the introduction of a saturable absorber (SA) allows one to suppress the nonsoliton part of the optical signal and to achieve ultralong stable soliton propagation [7,8]. In this paper we investigate the impact of a saturable absorber on the pulse transmission in links exploiting SMFs and in-line SOAs. We investigate the stability of the NLSE soliton propagation and determine a critical sliding rate in the system with saturated gain.

Optical pulse propagation down the optical transmission line is governed by the equation (see, e. g., [6])

$$i\frac{\partial E}{\partial Z}-\frac{\beta_2}{2}\frac{\partial^2 E}{\partial T^2}+\sigma|E|^2E=i\hat{G}E+i\hat{H}E. \qquad (1)$$

Here T is the retarded time; E is normalized such that $|E|^2$ represents optical power in W; β_2 is the chromatic dispersion parameter; $\sigma=(2\pi n_2)/(\lambda_0 A_{eff})$ is the nonlinear coefficient; there n_2 is the nonlinear refractive index, $\lambda_0=1.3$ μm is the carrier wavelength, and A_{eff} is the effective fiber area. Operator $\hat{H}=(1/2\pi)\sum_{k=1}^{N}\delta(Z-Z_k)\int[H(\omega)-1]E(\omega,Z)\times\exp(-i\omega t)d\omega$ describes filtering effect. Here $E(\omega,Z)=\int E(t,Z)\exp(i\omega t)dt$, the filter transfer function $H(\omega)=[1+i2(\omega-\omega_f)/B]^{-1}$ with a filter bandwidth B and ω_f varying along the fiber in the case of sliding filtering. In the parabolic approximation the filtering effect can be described as $\hat{H}E=c_1E+c_2(i\partial_t-\omega_f)^2E$. The operator $\hat{G}=-\gamma+\sum_{k=1}^{N}\delta(Z-Z_k)\{\exp[0.5(1-i\alpha_H)h(t)]-1\}$ represents the losses and periodic amplification by SOAs, $Z_k=kZ_a$ $(i=k,\ldots,N)$ are the semiconductor amplifier locations, the loss coefficient (in km^{-1}) $\gamma=0.05\ln(10)\alpha$ accounts for the fiber attenuation along an amplifier span; α is fiber loss in dB/km. The point action of the semiconductor amplifier is given under reasonable assumptions (see [9] for details) by $E_{out}=\exp[0.5(1-i\alpha_H)h(t)]E_{in}$. The gain coefficient is found from a solution of the equation

$$\frac{dh}{dt}=\frac{g_0L_a-h}{T_1}-[\exp(h)-1]\frac{|E_{in}|^2}{\varepsilon_{sat}}. \qquad (2)$$

Here g_0 is a small-signal gain, L_a is the amplifier length, T_1 is the spontaneous carrier lifetime, α_H is the Henry factor or the linewidth enhancement factor, respectively, and ε_{sat} is the saturation energy of the amplifier.

A typical set of practical parameters of the transmission line and SOA operating at 1.3 μm is the following: the chromatic dispersion parameter $\beta_2\approx-1$ ps^2/km; the nonlinear refractive index $n_2=2.6\times10^{-20}m^2$/W, the carrier wavelength $\lambda_0=1300$ nm; the effective fiber area $A_{eff}=74$ μm^2; the loss $\alpha=0.4$ dB/km; amplification distance $Z_a=50$ km; spontaneous lifetime $T_1=200$ ps; saturation energy $\varepsilon_{sat}=6$ pJ; Henry factor $\alpha_H=5$.

When the amplification distance Z_a is much smaller than the characteristic dispersion length $Z_D = t_0^2/|\beta_2|$, the averaged pulse propagation along such a transmission system can be described by a distributed model [2,3]:

$$i\frac{\partial A}{\partial z} + \frac{1}{2}\frac{\partial^2 A}{\partial t^2} + |A|^2 A = i\,\delta A + i\beta\frac{\partial^2 A}{\partial t^2} + tfA + is|A|^2 A - (i+\alpha_H) r A\int_{-\infty}^{t}|A|^2 d\tau. \qquad (3)$$

Here time is normalized to the characteristic pulse width t_0 [which is related to the full width at half maximum (FWHM) of a soliton by $t_{FWHM} \simeq 1.763 t_0$]; $|A|^2$ is normalized to the pulse input power P_0; z is normalized to the Z_D, $\delta = Z_D[\ln G_0/(2Z_a) - \gamma]$ is the excess gain; $G_0 = \exp(g_0 L_a)$ is the linear gain of the amplifier; $\beta = 2/(B^2 Z_a)$ is the distributed filtering strength, the term with f accounts for the sliding effect (see for details, e.g., [7]); gain saturation coefficient $r = (1 - G_0^{-1}) P_0 t_0/(2 z_a \varepsilon_{sat})$. We introduce here an additional term $s|A|^2 A$ modeling the action of the fast saturable absorber (see, e.g., [10,7,8]). In the parameter $s = \alpha_0 I_s \tau_f/\tau_s$, α_0 is the linear absorption, τ_f is the decomposition time of the exitons, τ_s the recombination time of free carriers, and I_s is the saturation intensity.

Equation (3) can be applied also to the problem of a pulse generation in the mode-locking laser systems using saturable absorber, dispersive, and Kerr elements [10,12,13]. For Eq. (3), in contrast to the NLSE, a stable solitonlike solution that presents a natural nonlinear mode of the system is not known. In the practical implementation an input signal in the form of Gaussian shape or the fundamental soliton (soliton of the NLSE) is usually used. Therefore, first, we consider how the soliton of the NLSE evolves in such a system in the practical case of small values of parameters on the right-hand-side of Eq. (3). Our aim is to find conditions under which propagation of the fundamental soliton can be stabilized even in the presence of perturbative terms on the right-hand side of Eq. (3). Following the standard technique (see, e.g., [7]) one can describe the adiabatic evolution of the NLSE soliton under the combined action of the filtering, amplification by SOA, and a saturable absorber, assuming slow z dependence of the soliton parameters,

$$A(z,t) = \eta \,\mathrm{sech}[\eta(t-t_0)]\exp[-ik(t-t_0) + i\phi(z)]. \qquad (4)$$

Evolution of the parameters η and k is given by

$$\frac{dk}{dz} = f - \frac{4}{3}\beta k\eta^2 - \frac{2}{3}\alpha_H r\eta^2, \qquad (5)$$

$$\frac{d\eta}{dz} = 2\eta\left[\delta - \beta k^2 - \frac{\eta^2}{3}(\beta - 2s) - r\eta\right]. \qquad (6)$$

Equilibrium states $(\tilde{\eta},\tilde{k})$ are solutions of the set of equations,

$$f = \frac{2\tilde{\eta}^2}{3}(2\beta\tilde{k} + \alpha_H r), \qquad (7)$$

$$\delta = \beta\tilde{k}^2 + \frac{\tilde{\eta}^2}{3}(\beta - 2s) + r\tilde{\eta}. \qquad (8)$$

To study the stability of the stationary points given by Eqs. (7) and (8) let us linearize Eqs. (5) and (6) on the background of stationary point $(\tilde{\eta},\tilde{k})$: $\eta = \tilde{\eta} + \delta\eta$, $k = \tilde{k} + \delta k$ with $\delta k \ll \tilde{k}$, $\delta\eta \ll \tilde{\eta}$ and assume that $\delta k, \delta\eta \sim \exp(-\lambda z)$. It is easy to find that the linear stability of the stationary points is determined by the positiveness of the eigenvalues λ satisfying the equation

$$(\lambda - \tfrac{4}{3}\beta\tilde{\eta}^2)[\lambda + 2\delta - 2\beta\tilde{k}^2 - 2\tilde{\eta}^2(\beta - 2s) - 4r\tilde{\eta}] - \tfrac{16}{3}\beta\tilde{\eta}^2\tilde{k}(\alpha_H r + 2\beta\tilde{k}) = 0. \qquad (9)$$

Solutions of this equation are given by

$$\lambda_{1,2} = \tilde{\eta}\{r + \tfrac{4}{3}\tilde{\eta}(\beta - s) \pm \sqrt{[r + \tfrac{4}{3}\tilde{\eta}(\beta - s)]^2 + \tfrac{16}{3}\beta\tilde{k}(\alpha_H r + 2\beta\tilde{k}) - \tfrac{8}{3}\beta\tilde{\eta}[r + \tfrac{2}{3}\tilde{\eta}(\beta - 2s)]}\}. \qquad (10)$$

Both roots are positive if

$$r + \tfrac{4}{3}\tilde{\eta}(\beta - s) > 0$$

and

$$4\beta^2\tilde{k}^2 + 2r\alpha_H\beta\tilde{k} - \tilde{\eta}\beta[r + \tfrac{2}{3}\tilde{\eta}(\beta - 2s)] < 0.$$

This leads to the following limiting condition on the sliding rate:

$$\alpha_H r - \sqrt{\alpha_H^2 r^2 + 4\beta\tilde{\eta}[r + \tfrac{2}{3}\tilde{\eta}(\beta - 2s)]} \leq 3f \leq \alpha_H r + \sqrt{\alpha_H^2 r^2 + 4\beta\tilde{\eta}[r + \tfrac{2}{3}\tilde{\eta}(\beta - 2s)]}\,. \qquad (11)$$

This inequality gives limits in which the sliding rate can vary to achieve stable soliton propagation. The obtained requirement generalizes the condition on the sliding rate found in Refs. [11] and [3] for the system without gain saturation and saturable absorber: $f \leq \beta\tilde{\eta}\sqrt{8/27}$.

Adjusting the sliding parameter f to the action of the SOA $(f = \frac{2}{3}\alpha_H r)$, it is possible to compensate exactly the frequency shift induced by SOA [3]. Stationary point with $\tilde{k} = 0$ and $\tilde{\eta} = 1$ corresponds to the soliton that does not change its parameters during propagation along the transmission line. It is easy to find that the stability condition $\lambda_{1,2} > 0$ can be satisfied, namely, $\lambda_1 = 4\beta/3 > 0$ and $\lambda_2 = 2r + \frac{4}{3}(\beta - 2s)$ is also positive if $4s < 2\beta + 3r$.

Excess gain is given by

$$\delta = r + \tfrac{1}{3}(\beta - 2s). \qquad (12)$$

From this one can see that the impact of the saturable absorber on the pulse dynamics is that SA reduces excess gain (parameter δ) which is responsible for the instability of the continuous waves (CWs). Note that, in contrast to the case of amplification by means of linear (nonsaturated) amplifiers, in the problem that we consider there exists a limitation on the value of the excess gain δ from below ($\delta \geqslant r/2$).

Stabilization of the soliton propagation by means of sliding filters and saturable absorber allows us to enchance significantly a transmission capacity of the communication lines using SMFs and SOAs. It should be pointed out that the instability of the NLSE soliton does not mean that in this range of parameters stable pulse propagation is impossible. Linear instability of the NLSE soliton can be a first stage of transition from the initial state to the natural mode of the system under consideration. We would like to note here some new possibilities introduced by the SA in the problem of the existence of the stable localized solution of Eq. (3).

Let us consider the evolution of the energy integral $P=\int_{-\infty}^{+\infty}|A|^2 dt$.

$$\frac{dP}{dz}=2\delta P-2\beta\int |A_t|^2 dt+2s\int |A|^4 dt - rP^2. \qquad (13)$$

Evidently, the energy balance ($dP/dz=0$) must be achieved on the stationary solution. The action of the SA corresponds to the additional positive term on the right-hand side of Eq. (13). The interesting issue which arises considering Eq. (13) is that in the presence of the SA balance is possible even without the restriction $\delta>0$ that was the only possibility to achieve the equilibrium in the case $s=0$. Thus, introduction of the SA allows the existence of a localized solution with suppressed instability of the CW.

To make this possibility clear, consider now the localized solution of Eq. (3) (with $f=0$) having the particular form

$$A(z,t)=A_0 \operatorname{sech}\left(\frac{(t-z/v)}{T_0}\right)\exp\left\{ i\left[az-\Delta\omega(t-z/v) - b\ln \cosh\left(\frac{(t-z/v)}{T_0}\right)\right]\right\}, \qquad (14)$$

where the soliton width T_0, amplitude A_0, velocity v, frequency shift $\Delta\omega$, and chirp parameter b are functions of δ, β, s, r and α_H.

$$\frac{1}{v}=\frac{2\beta(\alpha_H b-1)+b+\alpha_H}{2\beta(1+b^2)} r A_0^2 T_0^2, \qquad (15)$$

$$\Delta\omega=-\frac{b+\alpha_H}{2\beta(1+b^2)} r A_0^2 T_0^2, \qquad (16)$$

$$T_0^2=\frac{b+\beta(b^2-1)}{\delta-\beta\Delta\omega^2}, \qquad (17)$$

$$A_0^2 T_0^2=\frac{3b}{2}\times\frac{1+4\beta^2}{2\beta-s}, \qquad (18)$$

$$a=\frac{1+4\beta b-b^2}{2T_0^2}-\Delta\omega^2/2-\Delta\omega/v, \qquad (19)$$

with

$$b=-\frac{3}{2}\frac{(1+2\beta s)}{(2\beta-s)}\pm\sqrt{2+\left[\frac{3}{2}\frac{(1+2\beta s)}{(2\beta-s)}\right]^2}. \qquad (20)$$

Two evident requirements $A_0^2>0$ and $T_0^2>0$ determine regions of possible parameters for which solution in the form (14) exists. Because parameters β and s are positive, we consider only the first quadrant with $\beta\geqslant 0$ and $s\geqslant 0$ in the plane (β,s). There are three regions in the plane (β,s). The first region corresponds to the solutions with

$$2\beta-s<0, \quad b=\frac{3}{2}\times\frac{1+2\beta s}{s-2\beta}-\sqrt{2+\left[\frac{3}{2}\times\frac{1+2\beta s}{2\beta-s}\right]^2}>0,$$
$$\delta<\beta\Delta\omega^2. \qquad (21)$$

The second branch of solutions is in the region

$$2\beta>s>\frac{2\beta}{1+3\sqrt{1+4\beta^2}},$$
$$b=-\frac{3}{2}\times\frac{1+2\beta s}{2\beta-s}+\sqrt{2+\left[\frac{3}{2}\times\frac{1+2\beta s}{2\beta-s}\right]^2}<0,$$
$$\delta<\beta\Delta\omega^2. \qquad (22)$$

The third region is determined by the conditions

$$0<s<\frac{2\beta}{1+3\sqrt{1+4\beta^2}},$$
$$b=-\frac{3}{2}\times\frac{1+2\beta s}{2\beta-s}+\sqrt{2+\left[\frac{3}{2}\times\frac{1+2\beta s}{2\beta-s}\right]^2}<0,$$
$$\delta>\beta\Delta\omega^2. \qquad (23)$$

The obtained conditions are sufficient for existence of the solutions in the form (14). This solution is a generalization of the soliton found in Refs. [14] and [2] to the case of the system with saturable absorbers. This is the so-called autosoliton, which means that soliton parameters are determined by medium only [by the external parameters in Eq. (3)] in contrast to the NLSE soliton. The important new feature introduced by the SA is that the excess gain parameter δ is not *necessarily positive* for all types of the localized solution described above. There exist branches, given by Eqs. (21) and (22) with negative δ for which the instability of the CW is automatically suppressed. The curve $s=2\beta/(1+3\sqrt{1+4\beta^2})$ is a line of singularity separating solutions with $\delta-\beta\delta\omega^2>0$ and autosolitons with $\delta-\beta\delta\omega^2<0$. The description of the obtained solutions is very close to a classification of the solitons of the complex Ginzburg-Landau equation developed in Ref. [15]. The role of such a special singular line for the case of the complex Ginzburg-Landau equation was pointed out by Akhmediev and co-workers in Ref. [15] (see also [16]). We would like to note that the parameter r occurs in the equations determined autosoliton amplitude A_0 and width T_0 only in the construction $\delta-\beta\delta\omega^2$. Therefore, the singularity line

$s=2\beta/(1+3\sqrt{1+4\beta^2})$ coincides with a special curve found in Ref. [15]. Without the SA, an asymptotic solution can be stabilized by use of the sliding filters [2,3]. Forming an asymptotic pulse is asymmetric in time and cannot be described by the symmetric solution (14). It is interesting to compare an asymptotic pulse that emerges in the case $s=0$ with the dissipative soliton described in Ref. [13]. In [13] it was considered a particular case of Eq. (3) with $\beta=f=s=\alpha_H=0$ and it has been found that the dissipative soliton is an asymptotic state of the system. An asymptotic solution of Eq. (3) with $s=0$ will be analyzed in more detail in a forthcoming publication.

In conclusion, by perturbation theory we analyzed the stability of the optical soliton propagation in communication systems using SMFs, periodic amplification by SOAs, and soliton control by means of sliding filters and saturable absorbers. A critical sliding rate has been determined that allows stabilization of the soliton propagation along such a system. It was shown that exploitation of the saturable absorber can reduce excess gain, which is responsible for the instability of the CW. Introduction of the SA also allows existence of the soliton solution in the region of parameters where instability of the CW is suppressed.

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[1] J. J. Reid, C. T. H. F. Liendenbaum, L. F. Tiemeijer, A. J. Boot, P. I. Kuindersma, I. Gabitov, and A. Mattheus, in *Proceedings of the 20th Eur. Conf. Opt. Commun.*, Post-deadline papers (Instituto Internazionale delle Communicazioni, Via Pertinace—Villa Piaggio, Genova, Italia, 1994), pp. 61–64.

[2] A. Mecozzi, Opt. Lett. **20**, 1616 (1995).

[3] S. Wabnitz, Opt. Lett. **20**, 1975 (1995).

[4] F. Matera and M. Settembre, Microwave Opt. Technol. Lett. **10**, 207 (1995).

[5] F. Matera and M. Settembre, J. Lightwave Technol. **14**, 1 (1996).

[6] V. Haegele, A. Mattheus, R. Zengerle, I. Gabitov, and S. K. Turitsyn, J. Opt. Commun. (to be published).

[7] A. Hasegawa and Y. Kodama, *Solitons in Optical Communications* (Oxford University Press, New York, 1996).

[8] D. Atkinson, W. H. Loh, V. V. Afanasjev, A. B. Grudinin, A. J. Seeds, and D. N. Payne, Opt. Lett. **19**, 1514 (1994).

[9] G. P. Agrawal and N. A. Olsson, IEEE J. Quantum Electron. **25**, 2297 (1989).

[10] H. A. Haus and Y. Silberberg, J. Opt. Soc. Am. B **2,** 1237 (1985).

[11] L. F. Mollenauer, J. P. Gordon, and S. G. Evangelides, Opt. Lett. **17**, 1575 (1992); Y. Kodama and S. Wabnitz, *ibid.* **18**, 1311 (1993); **19**, 162 (1994).

[12] K. P. Komarov, Opt. Spektrosk. **60**, 379 (1986) [Opt. Spectrosc. **60**, 231 (1986)]; K. P. Komarov, A. S. Kych'yanov, and V. D. Ugozhaev, Opt. Commun. **57**, 279 (1986).

[13] E. V. Vanin, A. I. Korytin, A. M. Sergeev, D. Anderson, M. Lisak, and L. Vazquez, Phys. Rev. A **49**, 2806 (1994).

[14] V. S. Grigoryan and T. S. Muradyan, J. Opt. Soc. Am. B **8**, 1757 (1991); V. S. Grigoryan, A. I. Maimistov, and Yu. M. Sklyarov, Zh. Eksp. Teor. Fiz. **94**, 174 (1988) [Sov. Phys. JETP **67**, 530 (1988)].

[15] N. N. Akhmdediev and V. V. Afanasjev, Phys. Rev. Lett. **75**, 2320 (1995); N. N. Akhmediev, V. V. Afanasjev, and J. M. Soto-Crespo, Phys. Rev. E **53**, 1190 (1996).

[16] A. I. Chrenyukh and S. K. Turitsyn, Opt. Lett. **20**, 1 (1995).